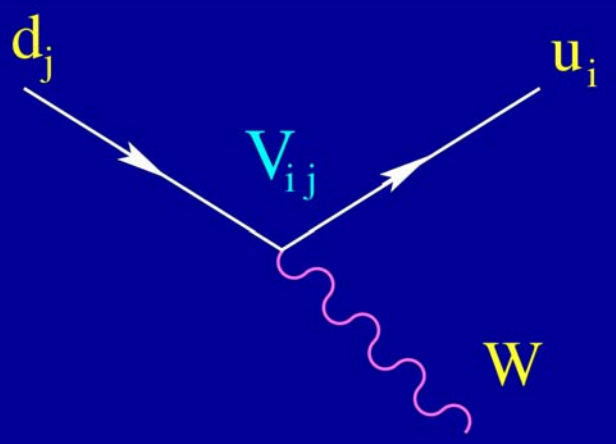
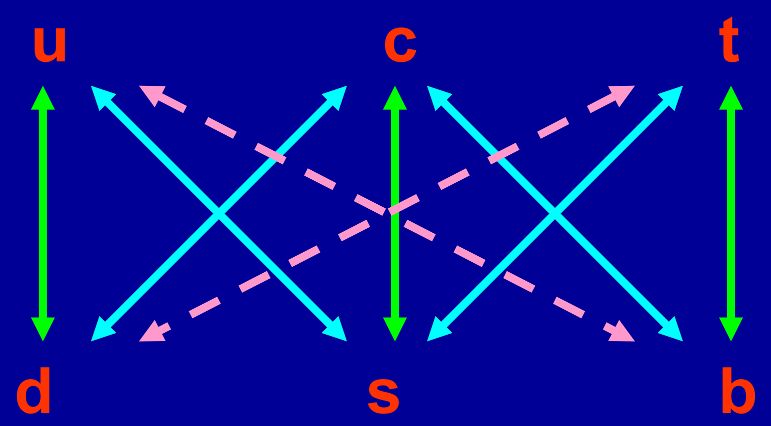
3. Higher-Order Transitions

- Structure of the CKM matrix
- Meson Mixing
- Rare Decays
- Indirect $\mathbf{V}_{ij}$ determinations

Flavour Changing Charged Currents

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij} d_j + \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}$$





QUARK MIXING MATRIX $\mathbf{V}_{ij}$

Cabibbo – Kobayashi – Maskawa (CKM)

V_{ij} DETERMINATIONS

CKM entry	Value	Source
$\lvert V_{ud}\rvert$	0.97377 ± 0.00027	Nuclear β decay
$\lvert V_{us}\rvert$	0.2230 ± 0.0025	$K_{e3}\,,\ K_{\mu 2}\,,\ \tau \to \nu_\tau S^-$
$\lvert V_{cd}\rvert$	0.224 ± 0.012	$\nu\, d \to c\, X$
$\lvert V_{cs}\rvert$	0.976 ± 0.014	$W^+ \to \text{had}\,,\ V_{uj}\,,\ V_{cd,cb}$
$\lvert V_{cb}\rvert$	0.0415 ± 0.0010	$B \to D^* l\, \bar{\nu}_l\ ,\ b \to c\, l\, \bar{\nu}_l$
$\lvert V_{ub}\rvert$	0.0043 ± 0.0003	$B \to \pi l\, \bar{\nu}_l\ ,\ b \to u\, l\, \bar{\nu}_l$
$\lvert V_{tb}\rvert \Big/ \sqrt{\sum_q \lvert V_{tq}\rvert^2}$	$0.97\,^{+\,0.16}_{-\,0.12}$	$t \to bW / qW$

$$\lvert \mathbf{V_{ud}} \rvert^2 + \lvert \mathbf{V_{us}} \rvert^2 + \lvert \mathbf{V_{ub}} \rvert^2 \;=\; \mathbf{0.9980 \pm 0.0017}$$

$$\sum_j \left(\lvert \mathbf{V_{uj}} \rvert^2 + \lvert \mathbf{V_{cj}} \rvert^2 \right) \;=\; \mathbf{1.999 \pm 0.025} \qquad \text{(LEP)}$$

$N_f = 3$: 3 angles, 1 phase (CKM)

$c_{ij} \equiv \cos\theta_{ij}$; $s_{ij} \equiv \sin\theta_{ij}$

$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

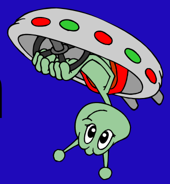
$$= \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\,\lambda^2 \\ A\lambda^3\,(1-\rho-i\,\eta) & -A\,\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$$s_{12} \equiv \lambda \quad , \quad s_{23} \equiv A\lambda^2 \quad , \quad s_{13}\, e^{-i\delta_{13}} \equiv A\lambda^3 \left(\rho - i\,\eta\right)$$

$\lambda \approx \sin\theta_C \approx 0.223$; $A \approx 0.83$; $\sqrt{\rho^2 + \eta^2} \approx 0.46$

$\delta_{13} \neq 0 \quad (\eta \neq 0)$ ⟹ $\not{CP}$

$P^0 - \bar{P}^0$ MIXING: General Description



$$|\psi(t)\rangle = a(t)\,|P^0\rangle + b(t)\,|\bar{P}^0\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \qquad , \qquad i\,\frac{d}{dt}\,|\psi(t)\rangle = \mathcal{M}\,|\psi(t)\rangle$$

$\mathcal{CPT}$:

$$\mathcal{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

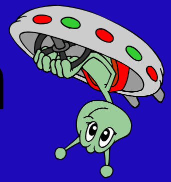
Dispersive:

$$M_{12} = \frac{\langle P^0|\mathcal{H}_{\Delta P=2}|\bar{P}^0\rangle}{2M_P} + \frac{1}{2M_P}\,\mathcal{P}\int ds\, \frac{\sum_X \int dQ_X\, \langle P^0|\mathcal{H}_{\Delta P=1}|X\rangle\,\langle X|\mathcal{H}_{\Delta P=1}|\bar{P}^0\rangle}{M_P^2 - s}$$

Absorptive:

$$\Gamma_{12} = \frac{\pi}{M_P}\sum_X \int dQ_X\, \delta(M_P^2 - s)\,\langle P^0|\mathcal{H}_{\Delta P=1}|X\rangle\,\langle X|\mathcal{H}_{\Delta P=1}|\bar{P}^0\rangle$$

$P^0 - \bar{P}^0$ MIXING: General Description



$\mathcal{CPT}$:

$$\mathcal{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

Eigenstates:

$$\langle P_- | P_+ \rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} \approx 2\,\mathrm{Re}\,(\bar{\varepsilon})$$

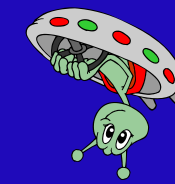
$$|P_\mp\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left[p\,|P^0\rangle \mp q\,|\bar{P}^0\rangle \right] \qquad , \qquad \frac{q}{p} \equiv \frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \right)^{1/2}$$

$\mathcal{CP}$ **Symmetry:** $M_{12} = M_{12}^*$, $\Gamma_{12} = \Gamma_{12}^*$ ➡ $|q/p| = 1$

➡ $$|P_{1,2}\rangle \equiv \frac{1}{\sqrt{2}} \left(|P^0\rangle \mp |\bar{P}^0\rangle \right) \qquad , \qquad \mathcal{CP}\,|P_{1,2}\rangle = \pm |P_{1,2}\rangle$$

[Phase convention: $\mathcal{CP}|P^0\rangle = -|\bar{P}^0\rangle$]

$P^0 - \bar{P}^0$ MIXING: Time Evolution



A P^0 / $\bar{P}^0$ state produced at t=0 evolves into

$$\begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} g_1(t) & \frac{q}{p}\, g_2(t) \\ \frac{p}{q}\, g_2(t) & g_1(t) \end{pmatrix} \begin{pmatrix} |P^0\rangle \\ |\bar{P}^0\rangle \end{pmatrix}$$

$$\begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix} = e^{-iMt}\, e^{-\Gamma t/2} \begin{pmatrix} \cos\left[(\Delta M - \frac{i}{2}\Delta\Gamma)t/2\right] \\ -i\,\sin\left[(\Delta M - \frac{i}{2}\Delta\Gamma)t/2\right] \end{pmatrix}$$

$$\Delta \mathrm{M} \equiv \mathrm{M}_{\mathrm{P}_+} - \mathrm{M}_{\mathrm{P}_-} \qquad\qquad \Delta\Gamma \equiv \Gamma_{\mathrm{P}_+} - \Gamma_{\mathrm{P}_-}$$

$$(\Delta M)^2 - \frac{1}{4}\,(\Delta\Gamma)^2 = 4\,|M_{12}|^2 - |\Gamma_{12}|^2 \qquad , \qquad \Delta M\,\Delta\Gamma = 4\,\mathrm{Re}\left(M_{12}\Gamma_{12}^*\right)$$

$$\Delta M \equiv M_{P_+} - M_{P_-} \qquad , \qquad \Delta\Gamma \equiv \Gamma_{P_+} - \Gamma_{P_-}$$

- **$K^0 \to 2\pi$, $K^0 \to 3\pi$** $\qquad \mathcal{CP}\,|\pi\pi\rangle = +\,|\pi\pi\rangle$

$$|K_S\rangle \equiv |K_-\rangle \approx |K_1\rangle + \bar\varepsilon_K\,|K_2\rangle \quad , \quad |K_L\rangle \equiv |K_+\rangle \approx |K_2\rangle + \bar\varepsilon_K\,|K_1\rangle$$

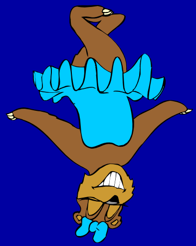
➡ $$\Gamma_{K_L} \ll \Gamma_{K_S}$$ $$\Delta\Gamma_{K^0} \approx -\Gamma_{K_S} \approx -2\,\Delta M_{K^0}$$

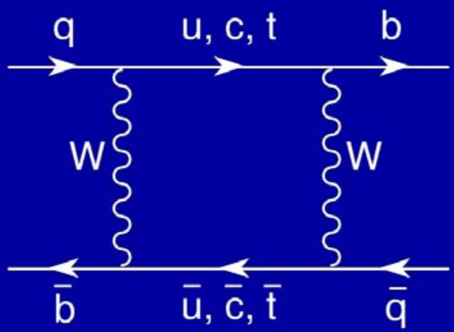
- **$B^0 \to X$** $\qquad$ Many open decay modes common to B^0 and $\bar{B}^0$

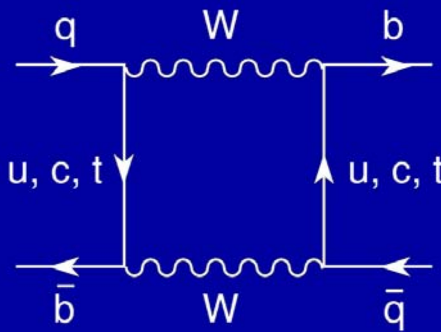
➡ $$\Delta\Gamma_{B^0} \ll \Gamma_{B^0}$$

$$\Delta\Gamma_{B^0}/\Delta M_{B^0} \sim m_b^2/m_t^2 \ll 1$$

$B^0 - \bar{B}^0$ MIXING







GIM: $\left(\sum_{i=u,c,t} \mathbf{V}_{i,d}\, \mathbf{V}^*_{i,b} = 0 \right)$ ⟹ **Mixing $\sim$ $m_t - m_c$, $m_t - m_u$**

$V_{ud}\, V^*_{ub} \sim V_{cd}\, V^*_{cb} \sim V_{td}\, V^*_{tb} \sim A\lambda^3$ ⟹ **Top contribution dominates ΔM_B**

$\Delta\Gamma_B$ governed by c/u contributions (b decays) ⟹ $\Delta\Gamma_{B^0} / \Delta M_{B^0} \sim m_b^2 / m_t^2 \ll 1$

$$\left\langle \bar{B}^0 \middle| H \middle| B^0 \right\rangle \sim \left| V_{td} \right|^2 S(r_t, r_t) \left(\frac{4}{3} M_B^2\, f_B^2 \right) \hat{B}_B \qquad r_i \equiv m_i^2 / M_W^2 \quad (i = u, c, t)$$

$$\langle \bar{B}^0 \,|\, (\bar{b}\gamma^\mu (1-\gamma_5) d)\, (\bar{b}\gamma_\mu (1-\gamma_5) d) \,|\, B^0 \rangle \equiv \frac{8}{3} M_B^2 \,(\sqrt{2}\, f_B)^2\, \hat{B}_B$$

$B^0 - \bar{B}^0$ MIXING

Time-dependent probability:

$$\text{Prob}[B^0 \to \bar{B}^0](t) = \frac{1}{2}\, e^{-\Gamma_{B^0} t} \left[\cosh\left(\frac{\Delta\Gamma_{B^0}}{2}\, t\right) - \cos\left(\Delta M_{B^0}\, t\right)\right] \left|\frac{q}{p}\right|^2$$

$$\approx \frac{1}{2}\, e^{-\Gamma_{B^0} t} \left[1 - \cos\left(\Delta M_{B^0}\, t\right)\right] \qquad (\Delta\Gamma_{B^0}/\Delta M_{B^0} \ll 1)$$

Time-integrated probability:

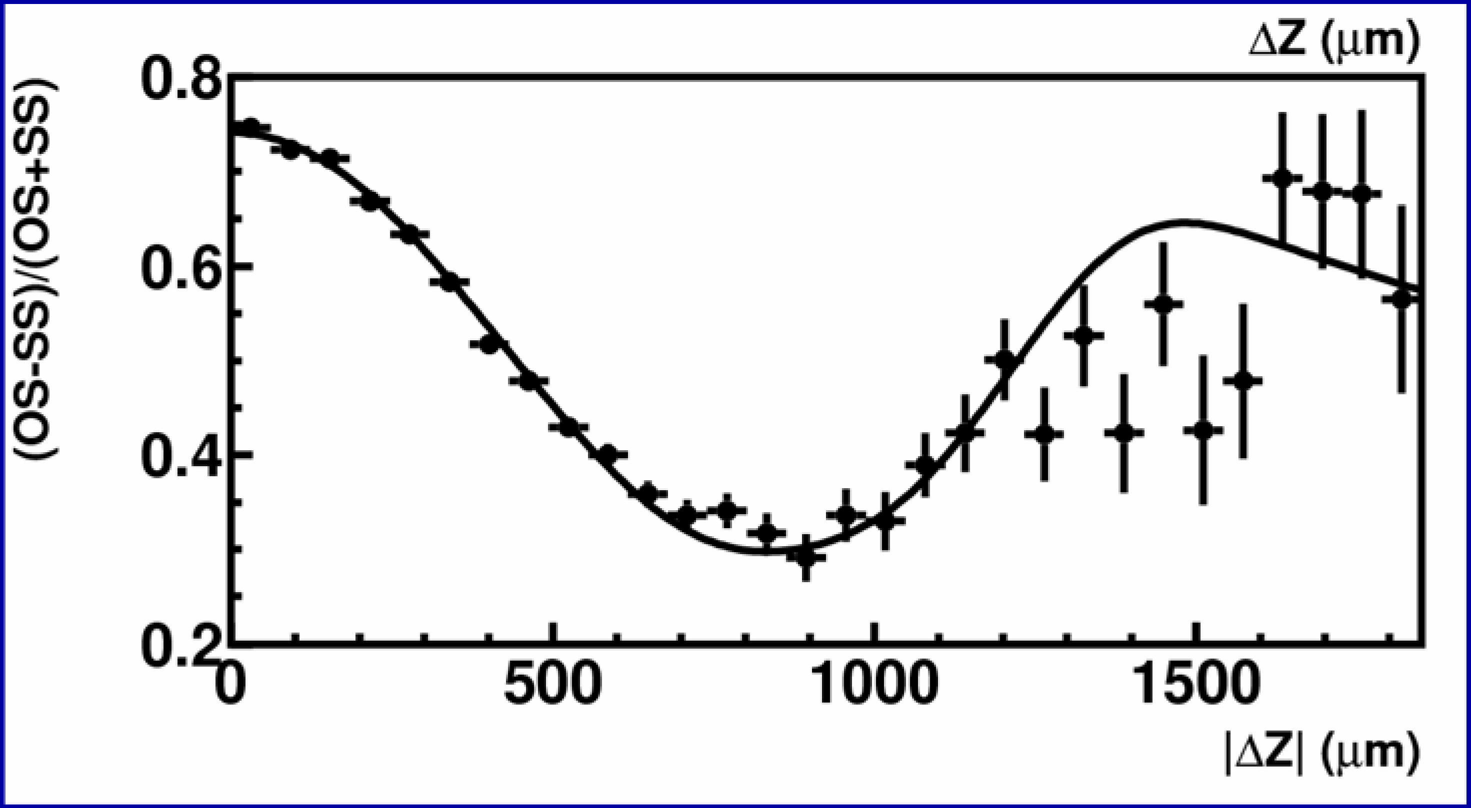
$$\chi \equiv \text{Prob}[B^0 \to \bar{B}^0] = \frac{x_d^2 + y_d^2}{2\,(1 + x_d^2)} \quad , \quad x_d \equiv \frac{\Delta M_{B^0}}{\Gamma_{B^0}} \quad , \quad y_d \equiv \frac{\Delta\Gamma_{B^0}}{2\,\Gamma_{B^0}}$$

B-flavour identification needed both at production and decay times

Flavour-specific decays (tagging): $B^0\,(\bar{b}) \to X\, l^+ \nu_l \quad , \quad \bar{B}^0\,(b) \to X\, l^- \nu_l$

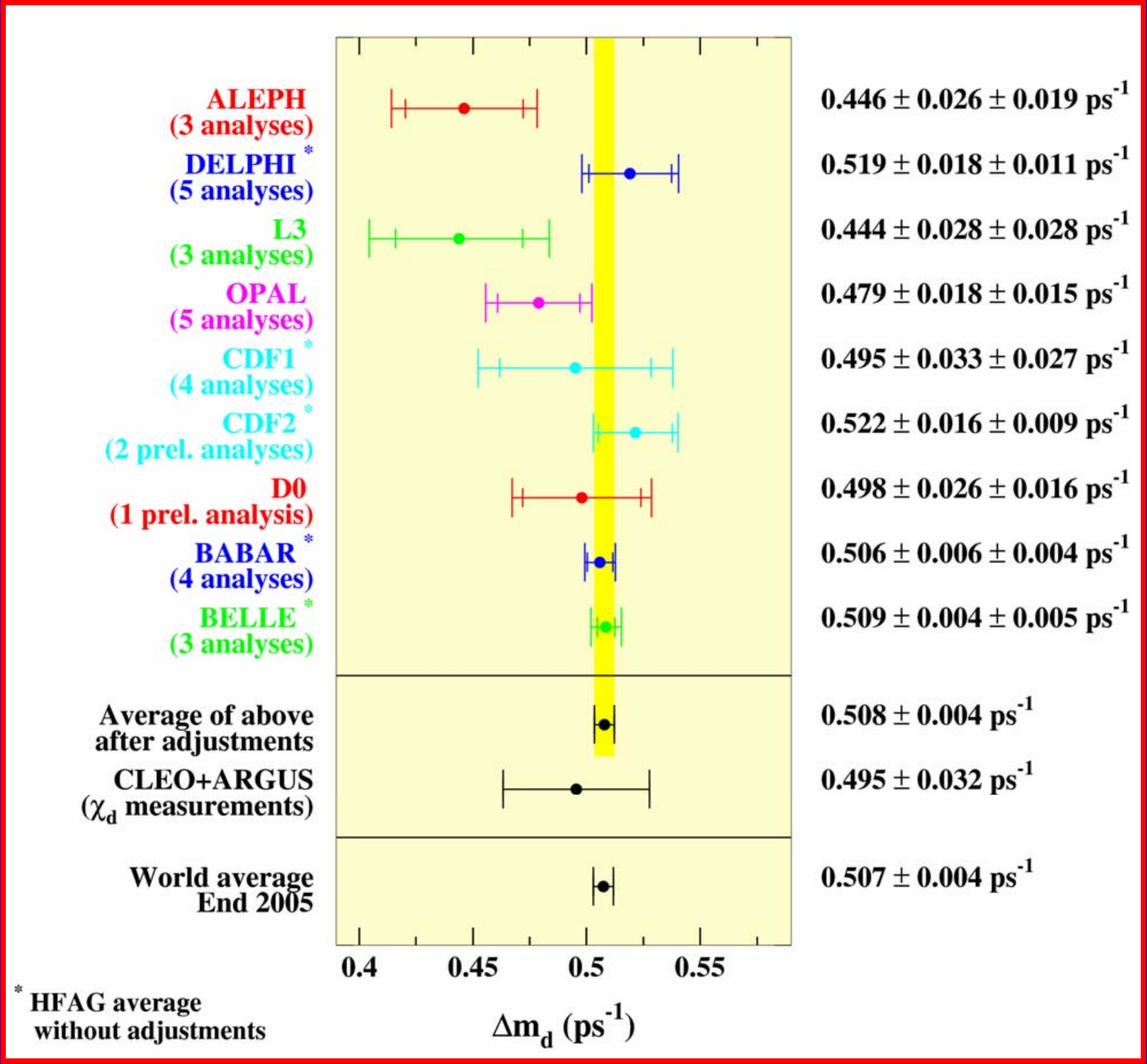
$$e^+ e^- \to B^0 \bar{B}^0 \to (X l \nu_l)\,(Y l \nu_l) \quad , \quad R_{ll} \equiv \frac{N(l^\pm l^\pm)}{N(l^\pm l^\mp) + N(l^\pm l^\pm)}$$

$$e^+e^- \to B^0\bar{B}^0 \to (X l \nu_l)\,(Y l \nu_l) \qquad , \qquad R_{ll} \equiv \frac{N(l^\pm l^\mp) - N(l^\pm l^\pm)}{N(l^\pm l^\mp) + N(l^\pm l^\pm)}$$

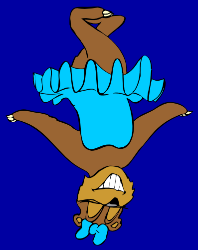


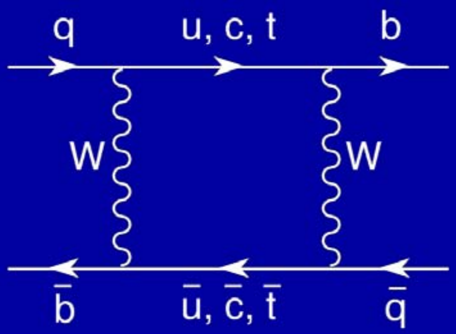
BELLE

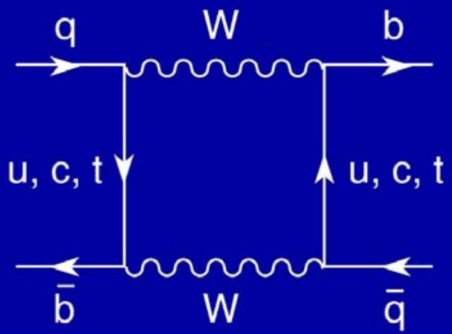
$$\Delta M_{B_d^0} = (0.507 \pm 0.004)\ \mathrm{ps}^{-1}$$



$B^0 - \bar{B}^0$ MIXING







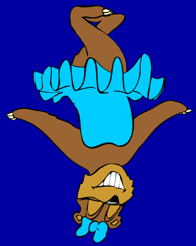
$$\left\langle \bar{B}^0 \middle| H \middle| B^0 \right\rangle \sim \left| V_{td} \right|^2 S(r_t, r_t) \left(\frac{4}{3} M_B^2 f_B^2 \right) \hat{B}_B \qquad r_i \equiv m_i^2 / M_W^2 \qquad (i = u, c, t)$$

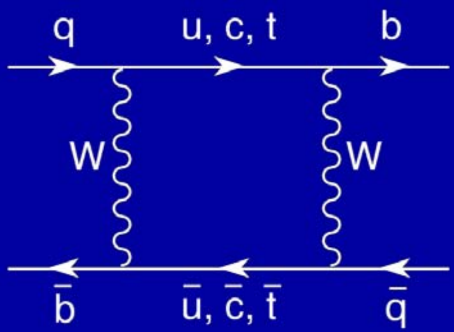
$$\langle \bar{B}^0 \,|\, (\bar{b}\gamma^\mu (1-\gamma_5) d)\, (\bar{b}\gamma_\mu (1-\gamma_5) d) \,|\, B^0 \rangle \equiv \frac{8}{3} M_B^2 \, (\sqrt{2}\, f_B)^2 \, \hat{B}_B$$

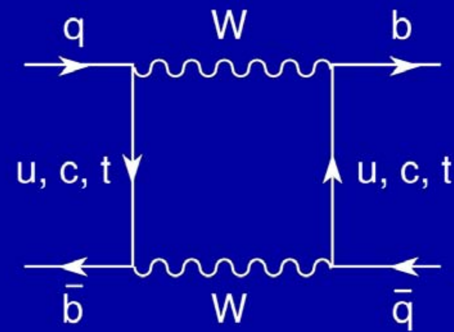
Lattice: $\sqrt{2}\, f_B \sqrt{\hat{B}_B} = (224 \pm 35)\ \mathrm{MeV}$

$$\Delta M_{B_d^0} = (0.507 \pm 0.004)\ \mathrm{ps}^{-1} \quad \Rightarrow \quad \left| V_{td} \right| = (8.1 \pm 1.3) \times 10^{-3}$$

$B^0 - \bar{B}^0$ MIXING



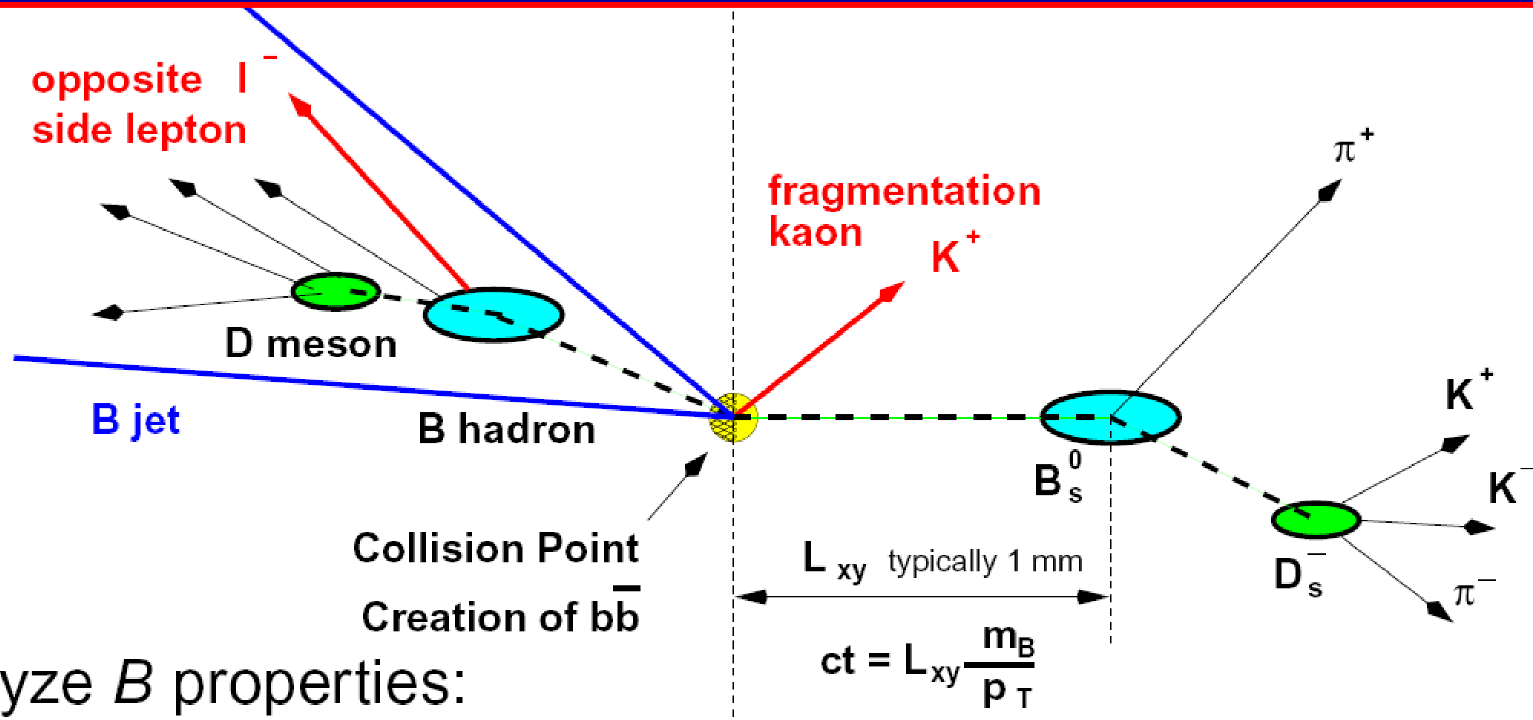




$$\Delta M_{B_d^0} = (0.507 \pm 0.004)\ \mathrm{ps}^{-1} \quad \Rightarrow \quad |V_{td}|$$

- $\Delta M_{B_d^0} / \Gamma_{B_d^0} = 0.775 \pm 0.008$
- $\Delta M_{B_s^0} = (17.31\,^{+0.33}_{-0.18} \pm 0.07)\ \mathrm{ps}^{-1}$ (CDF 2006) $\Rightarrow$ $|V_{ts}|^2 \gg |V_{td}|^2$
- $\Delta \Gamma_{B_s^0} / \Gamma_{B_s^0} = 0.31^{+0.10}_{-0.11}$ $\qquad \Delta\Gamma_{B^0} / \Delta M_{B^0} \sim m_b^2 / m_t^2 \ll 1$
- $|q/p|_{B_d^0} = 1.0015 \pm 0.0039$ $\qquad |q/p| - 1 \sim m_c^2 / m_t^2$

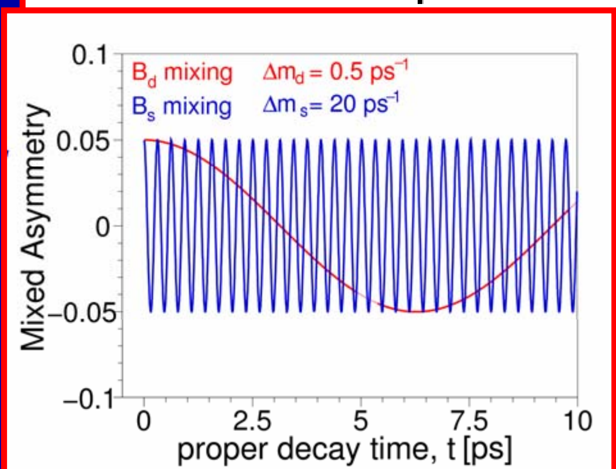
$\not{CP}$ very small



Analyze *B* properties:

+ flavor at decay: from final state
+ proper decay length *ct*: in *B* rest frame
+ flavor at production: from flavor tagging

(mass, etc): minimize backgrounds



4, G. Gómez-Ceballos, April 2006, FPCP, Vancouver, Canada

Fourier analysis

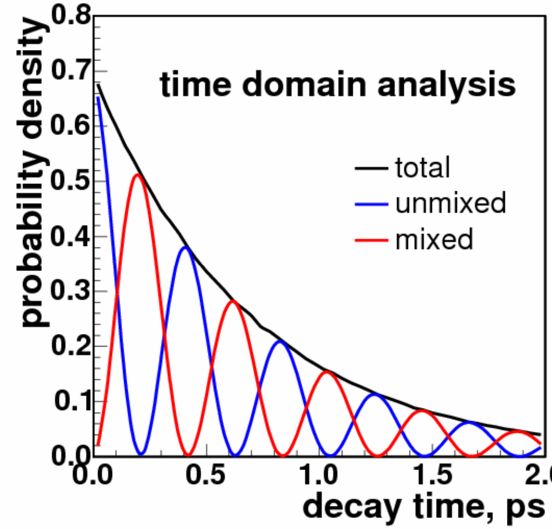
Two domains to fit for oscillation:

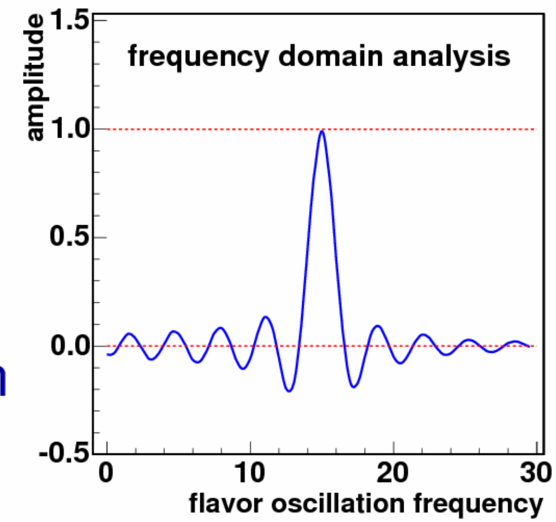
Time domain:

- ☞ fit for Δm_s in $P(t) \sim (1 \pm D \cos \Delta m_s t)$

Frequency domain: amplitude scan

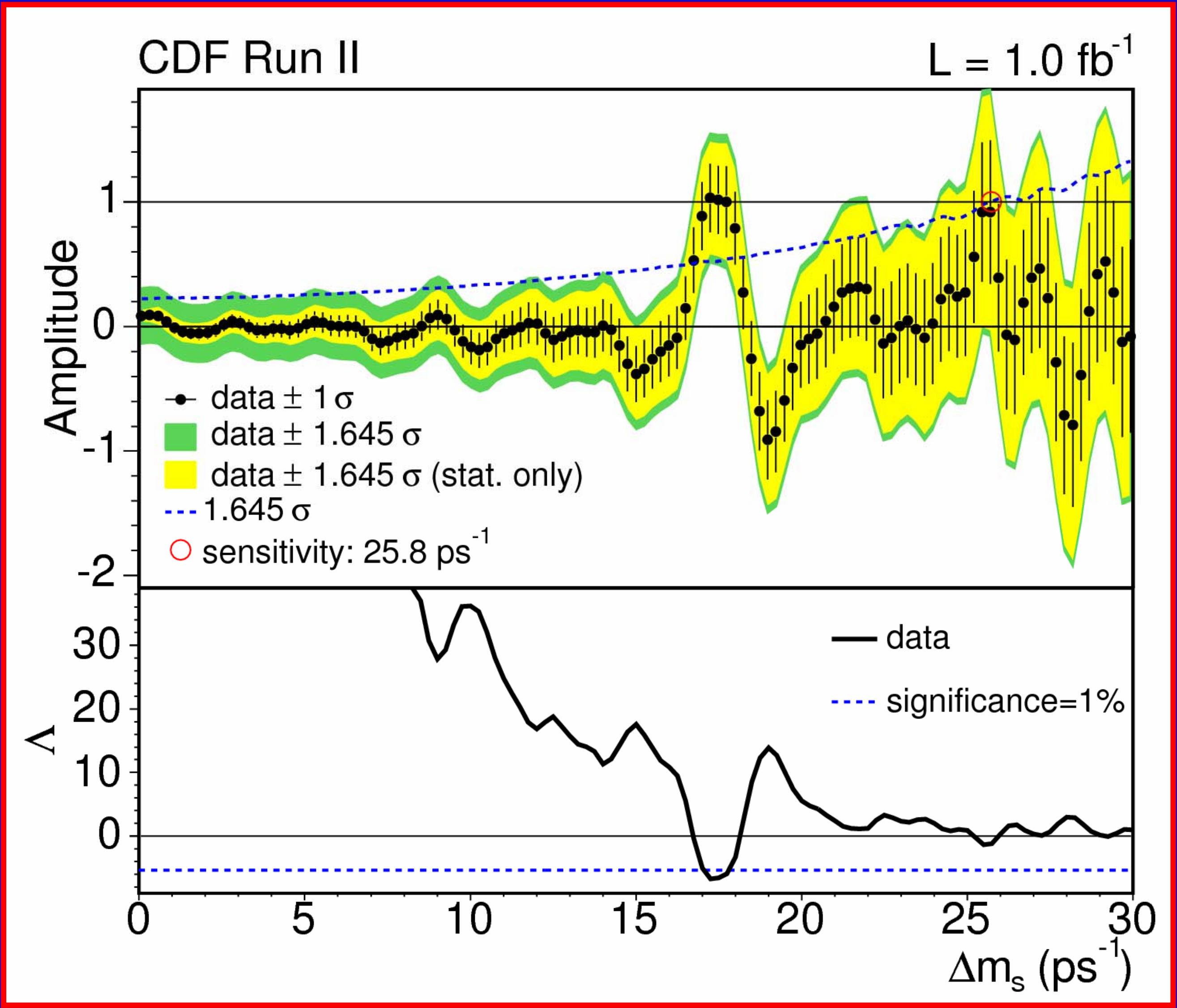
- ☞ introduce amplitude:
  $P(t) \sim (1 \pm \mathcal{A}\, D \cos \Delta m_s t)$
- ☞ fit for $\mathcal{A}$ at different Δm_s
- $\Rightarrow$ obtain frequency spectrum
- ☞ true $\Delta m_s \Rightarrow \mathcal{A} = 1$, else $\mathcal{A} = 0$
- ☞ traditionally used for B_s^0 mixing search
- $\Rightarrow$ easy to combine experiments





22, G. Gómez-Ceballos, April 2006, FPCP, Vancouver, Canada

$$\Delta M_{B_s^0} = (17.31 \,^{+0.33}_{-0.18} \pm 0.07)\ \text{ps}^{-1}$$

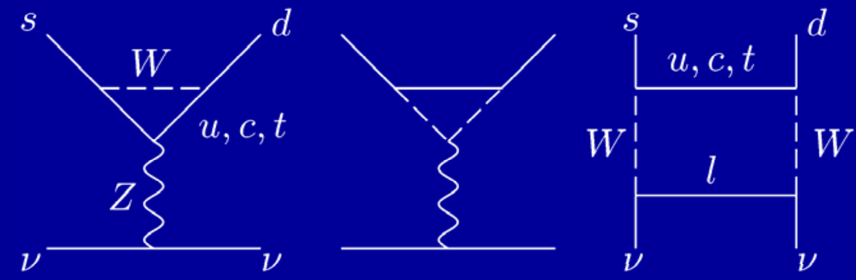


$$\frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}} = 1.21\,^{+0.047}_{-0.035}$$

$$\frac{|V_{td}|}{|V_{ts}|} = 0.208\,^{+0.001}_{-0.002}\,^{+0.008}_{-0.006}$$

(exp) (th)

$K \to \pi \nu \bar{\nu}$



$$\mathbf{T} \sim F\left(V_{is}^{*} V_{id}\,, m_i^2/M_W^2\right)\,\left(\bar{\nu}_L \gamma_\mu \nu_L\right)\,\left\langle \pi \middle| \bar{s}_L \gamma_\mu d_L \middle| K \right\rangle$$

$$\mathrm{Br}\left(K^+ \to \pi^+ \nu \bar{\nu}\right) = (8.0 \pm 1.1) \times 10^{-11} \sim A^4 \left[\eta^2 + (1.4 - \rho)^2\right]$$

$$\mathrm{Br}\left(K_L \to \pi^0 \nu \bar{\nu}\right) = (2.8 \pm 0.4) \times 10^{-11} \sim A^4\, \eta^2$$

Buras et al

Long-distance contributions are negligible

$$\mathbf{T}\left(K_L \to \pi^0 \nu \bar{\nu}\right) \neq 0 \quad \Rightarrow \quad \not{C}\!\not{P}$$

- **BNL:** few events! ⇒ $\mathrm{Br}\left(K^+ \to \pi^+ \nu \bar{\nu}\right) \sim 10^{-10}$
- **KTEV:** $\mathrm{Br}\left(K_L \to \pi^0 \nu \bar{\nu}\right) < 5.9 \times 10^{-7}$ (90% C.L.)

New Experiments Needed