PARTICLE COSMOLOGY 5

Anisotropies
The cosmological parameters
Inflation

Conformal time

$$\boxed{d\tau = \frac{dt}{a}}$$

$$ds^2 = -a^2(\tau)\left[d\tau^2 - \frac{dr^2}{1-kr^2} - r^2 d\Omega^2\right]$$

$$\text{MD}: \quad a \propto t^{2/3} \;\rightarrow\; \tau \propto a^{1/2} \propto \frac{1}{T^{1/2}}$$

Particle Horizon

Physical distance photons can travel until a time t

Defines causal distance at a given time t

in dt' light travels $c\,dt'$

today expanded to $c\,dt'\,\frac{a(t_0)}{a(t')}$

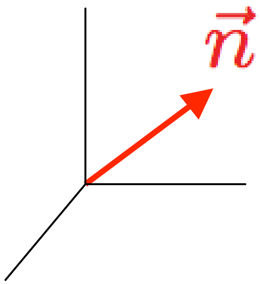
$$d_H(t_0) = a(t_0) \int_0^{t_0} \frac{dt'}{a(t')}$$

$$d_H(\tau) = a(\tau) \int_{\tau_0}^{\tau} d\tau' \qquad \tau_0 \leftrightarrow t_0$$

MD and RD : $\quad a \propto t^n \;\rightarrow\; d_H = \frac{n}{1-n} H^{-1}$

Correlations

Average $T_0 = 2.7\,\mathrm{K}$



anisotropy

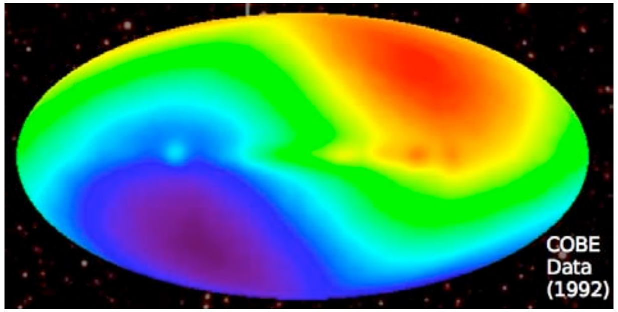
$$\Delta T(\vec{n}) = T(\vec{n}) - T_0 \qquad |\vec{n}| = 1$$

Two-point correlation

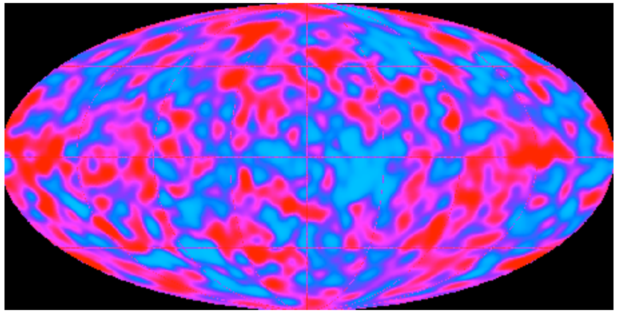
$$C(\theta) = \left\langle \left(\frac{\Delta T(\vec{n})}{T_0}\right)\left(\frac{\Delta T(\vec{m})}{T_0}\right) \right\rangle_{\vec{n}\cdot\vec{m}=\cos\theta}$$

$$= \frac{1}{4\pi}\sum(2l+1)\,C_l\,P_l(\cos\theta)$$

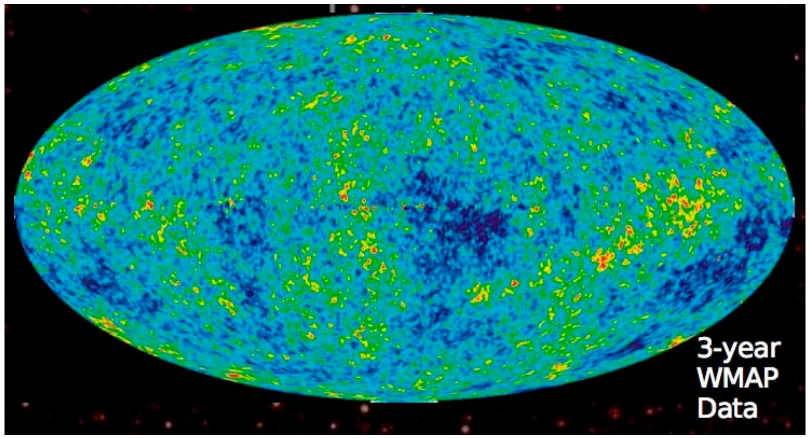
From COBE to MAP



dipole



Maximum fluctuation
75 \mu K



Correlations observed

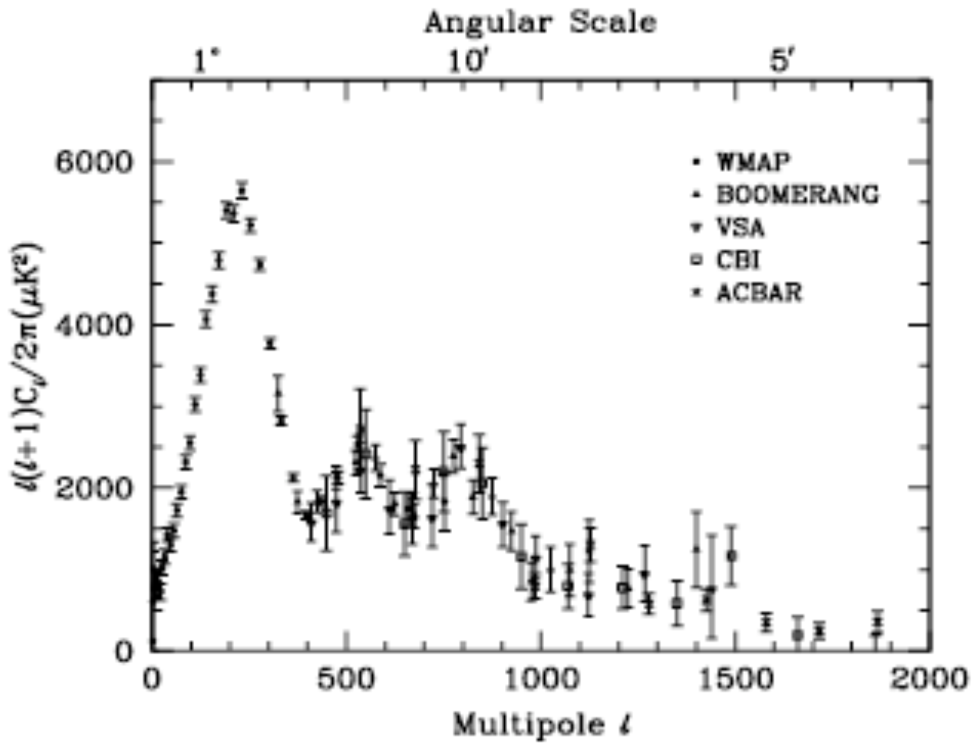


Figure 23.2: Band-power estimates from the *WMAP*, BOOMERANG, VSA, CBI, and ACBAR experiments. We have suppressed some of the low-ℓ and high-ℓ band-powers which have large error bars. Note also that the widths of the ℓ-bands varies between experiments. This plot represent only a selection of available experimental results, with some other data-sets being of similar quality. The multipole axis here is linear, so the Sachs-Wolfe plateau is hard to see. However, the acoustic peaks and damping region are very clearly observed, with no need for a theoretical curve to guide the eye.

Correlations expected

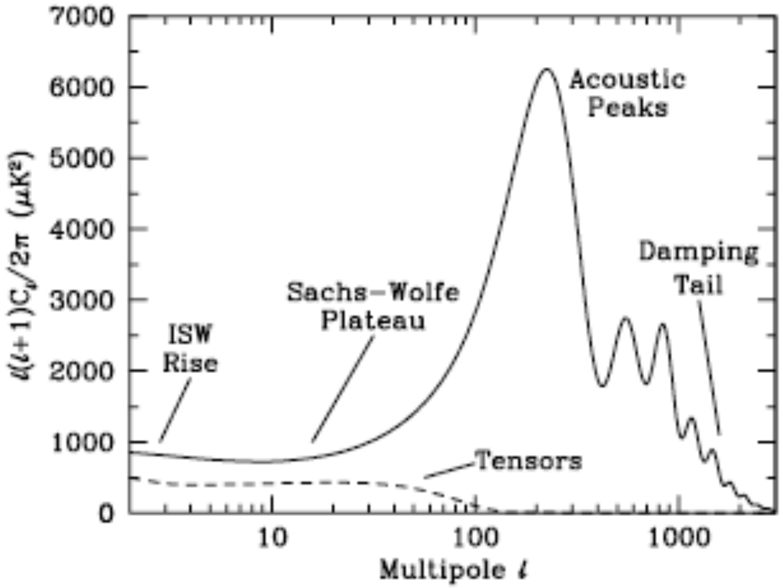


Figure 23.1: The theoretical CMB anisotropy power spectrum, using a standard ΛCDM model from CMBFAST. The x-axis is logarithmic here. The regions, each covering roughly a decade in ℓ, are labeled as in the text: the ISW Rise; Sachs-Wolfe Plateau; Acoustic Peaks; and Damping Tail. Also shown is the shape of the tensor (gravity wave) contribution, with an arbitrary normalization.

★ for $\theta > 1^o$

$$\left(\frac{\Delta T}{T}\right)_\theta \approx \left(\frac{\Delta \rho}{\rho}\right)_\lambda$$

$$\lambda \sim 100 h^{-1}\,\text{Mpc}\,\left(\frac{\theta}{\text{deg}}\right)$$

★ for $\theta < 1^o$

Acoustic oscillations

Acoustic horizon

Last-scattering surface is at

(in comoving coor.)

$$\int_{t_{LS}}^{t_0} \frac{dt}{a} = \int_{\tau_{LS}}^{\tau_0} d\tau = \tau_0 - \tau_{LS}$$

Sound velocity

Consider comoving scale of sound horizon at LS

$$c_s \tau_{LS}$$

Projection on the LS surface:

$$\theta_{AC} \simeq \frac{c_s \tau_{LS}}{\tau_0 - \tau_{LS}} \simeq \frac{\frac{1}{\sqrt{3}} \tau_{LS}}{\tau_0} \simeq \frac{1}{\sqrt{3}} \left(\frac{T_0}{T_{LS}} \right)^{1/2} \simeq 2^o$$

Vive legendre !

1752-1833

To see "structure" at $\Delta\theta < a$

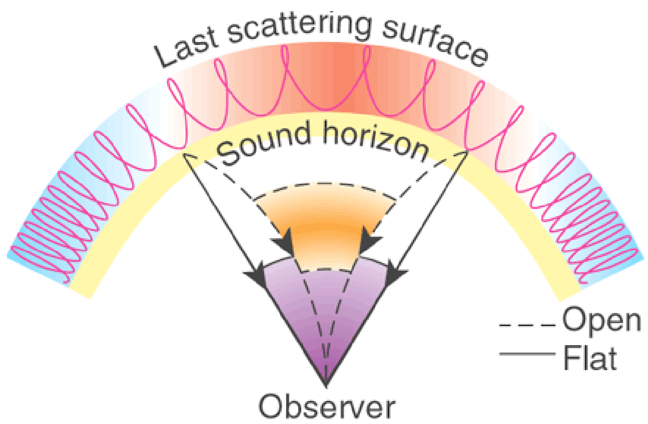
include:

$$P_0(\cos\theta), P_1(\cos\theta), \ldots, P_{l_{max}}(\cos\theta)$$

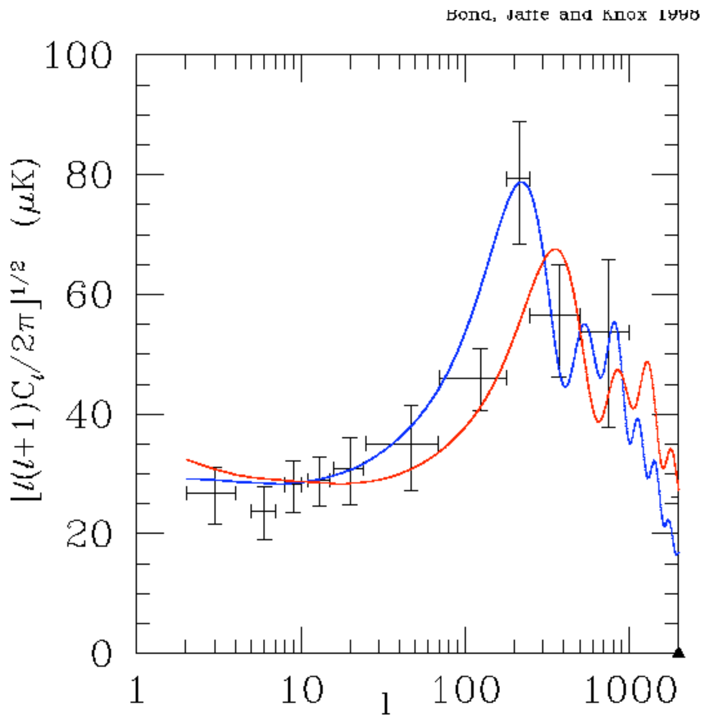
with $l_{max} \simeq \frac{\pi}{a}$

$$l_{AC} \simeq \frac{200}{\theta_{AC}\,(\text{deg})} \simeq 200$$

The first peak



Open: looks smaller



flat

open

$$l_{\rm peak} \sim \frac{200}{\Omega^{1/2}}$$

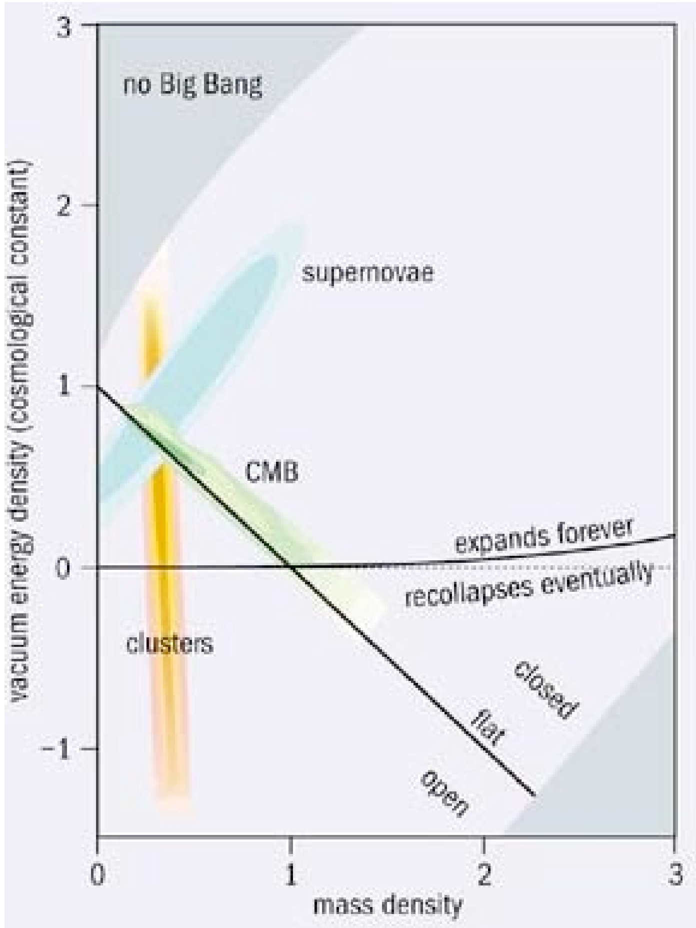
Cosmological parameters from CMB

Table 11: Joint Data Set Constraints on Geometry and Vacuum Energy

Data Set	Ω_K	Ω_Λ
WMAP + $h = 0.72 \pm 0.08$	$-0.003^{+0.013}_{-0.017}$	$0.758^{+0.035}_{-0.058}$
WMAP + SDSS	$-0.037^{+0.021}_{-0.015}$	$0.650^{+0.055}_{-0.048}$
WMAP + 2dFGRS	$-0.0057^{+0.0061}_{-0.0088}$	$0.739^{+0.026}_{-0.029}$
WMAP + SDSS LRG	$-0.010^{+0.011}_{-0.015}$	$0.728^{+0.020}_{-0.028}$
WMAP + SNLS	$-0.015^{+0.020}_{-0.016}$	$0.719^{+0.021}_{-0.029}$
WMAP + SNGold	$-0.017^{+0.022}_{-0.017}$	$0.703^{+0.030}_{-0.038}$

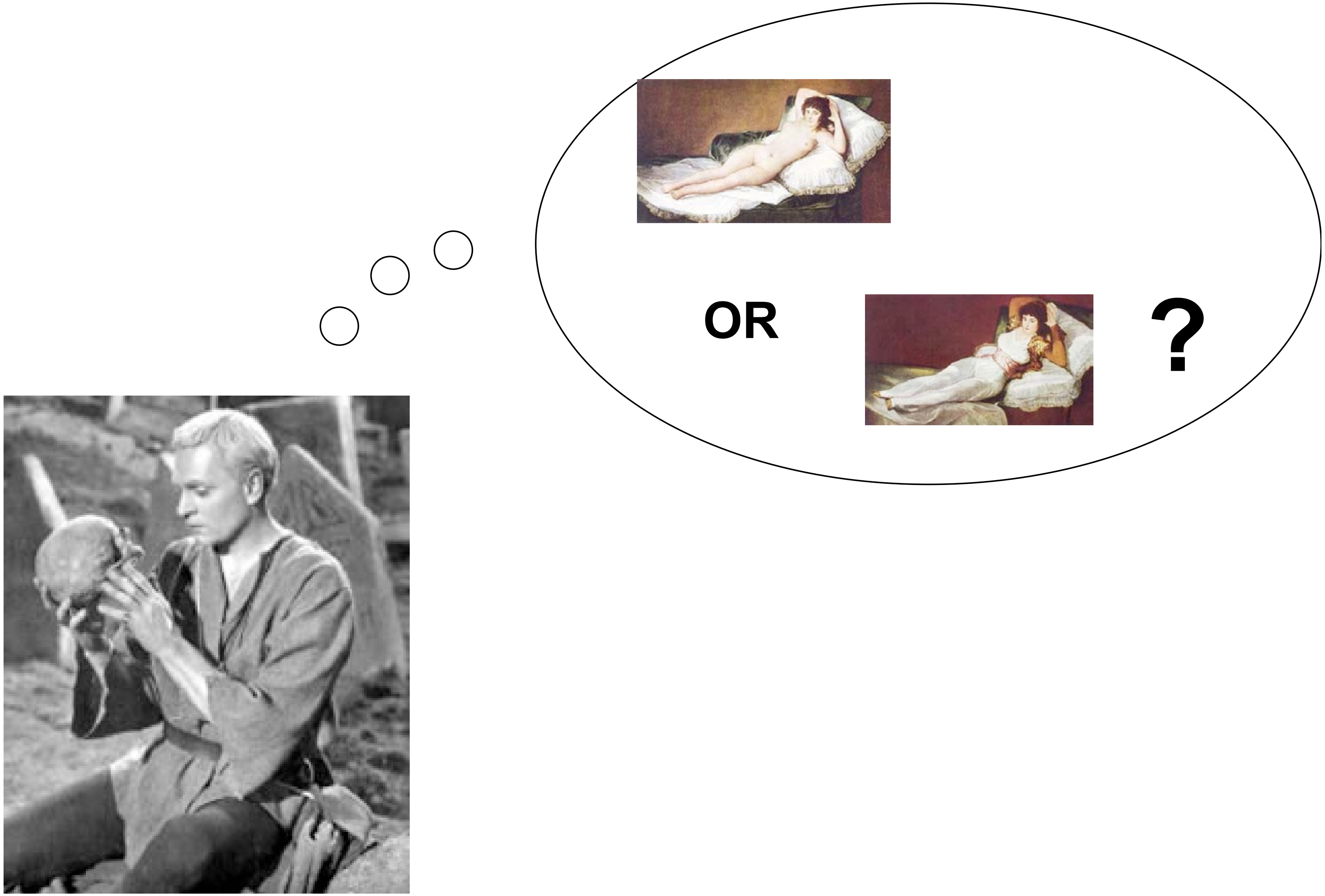
Notice priors

The bottom line



The concordance model

Doubt



The flatness problem (I)

Rewrite Friedmann eq. as

$$\Omega - 1 = \frac{k}{a^2 H^2}$$

$$\Omega = \frac{\rho_{\rm r} + \rho_{\rm m} + \rho_\Lambda}{\rho_{\rm crit}}$$

Spatial flatness

$$R_{\rm cur} = \frac{H^{-1}}{|\Omega - 1|^{1/2}}$$

$$H^2 = \ldots \; - \frac{k}{a^2} = \ldots \; \pm \frac{1}{R_{\rm cur}^2}$$

The flatness problem (II)

$$\Omega - 1 = \frac{k}{a^2 H^2}$$

$$|\Omega - 1| \propto \begin{cases} a^2 & \text{RD } (H^2 \propto a^{-4}) \\ a & \text{MD } (H^2 \propto a^{-3}) \end{cases}$$

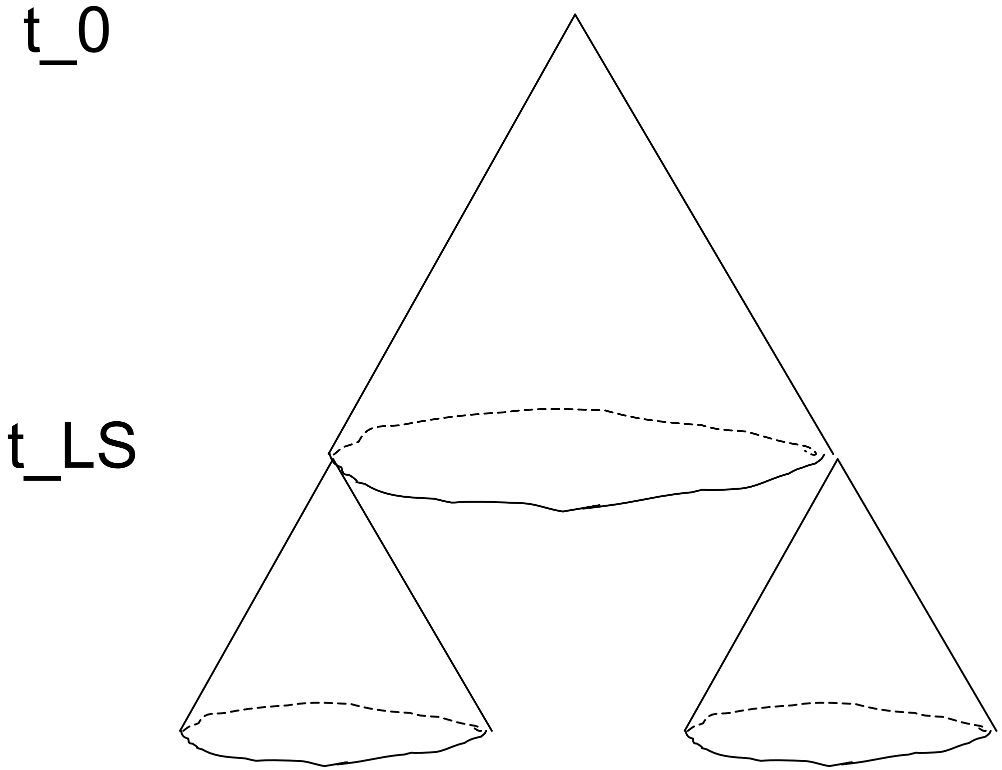
$$\frac{|\Omega_i - 1|}{|\Omega_0 - 1|} \sim \left(\frac{T_i}{T_{\rm dec}}\right)^2 \left(\frac{T_{\rm dec}}{T_0}\right) = \frac{T_i^2}{T_{\rm dec} T_0}$$

Wish $|\Omega_0 - 1| < O(1)$

Need $|\Omega_{\rm GUT} - 1| < O(10^{-53})$

$\Omega = 1$ unstable

The horizon problem



Acausal volumens in our present Hubble volume $\simeq \left(\frac{360^o}{2^o}\right)^3 \simeq 10^7$

Inflation

Period with $\ddot{a} > 0$

Simplest example: (cosm. ct.)

$$t_1 \rightarrow t_2 \quad \text{with} \quad H = \text{ct.}$$

$$|\Omega - 1| \propto e^{-2\,(t_2 - t_1)H}$$

$$(t_2 - t_1)H > 60$$